THE EFFECT OF LUMPED MASS MATRIX IN EXPLICIT FEM

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Explicit finite element based crash analysis codes have been successfully used to simulate real world vehicle crash events, such as frontal impact, side impact, roof crush, and rear impact. Computer analysis of vehicle crashworthiness has now become a powerful and effective design tool to bring new vehicles to market that meet both corporate and government safety requirements. Major automotive companies have built finite element simulation into vehicle design process. This increasing reliance on simulation has been made possible due to the successful development of nonlinear finite element methods along with the availability of affordable but powerful computers. To have a robust and reliable FEA tool, the verification and validation for the finite element methods becomes more and more inportant [1].

The weak form of updated Lagrangian formulation (principle of virtual power) has been used in explicit finite element method. For transient dynamics analysis, consistent mass matrix, or lumped mass matrix or the combination of these two can be used. The diagonal mass matrix, as one of the key component of the software, can be obtained by lumping the consistent mass matrix by row summation [2]. The inverse of a consistent mass matrix is impracticale in explicit finite element software. As a result, the effect of lumped mass matrix in crash worthiness analysis is still one of the key issues to be investigated. In this paper, we present an approximation method [3], to analyze the effect of mass matrices for highly nonlinear crashworthiness analysis. The combined mass matrix and its inverse are approximated as follows,

$$\boldsymbol{M} = \alpha \boldsymbol{M}_C + (1-\alpha)\boldsymbol{M}_D = \boldsymbol{M}_D\left[\boldsymbol{I} + \alpha \boldsymbol{M}_D^{-1}\left(\boldsymbol{M}_C - \boldsymbol{M}_D\right)\right]$$

$$\boldsymbol{M}^{-1} \cong \left[\boldsymbol{I} - \alpha \boldsymbol{M}_D^{-1}\left(\boldsymbol{M}_C - \boldsymbol{M}_D\right)\right]\boldsymbol{M}_D^{-1} = \boldsymbol{M}_D^{-1} - \alpha \boldsymbol{M}_D^{-1}\left(\boldsymbol{M}_C - \boldsymbol{M}_D\right)\boldsymbol{M}_D^{-1}$$

Examples of component crash with refined meshes are used for this study. Prelimary results showed that the lumped mass matrix is more stable than the consistent mass matrix. It also showed that with a small α, results from the combined mass matrix and diagonal mass matrix are close, whereas for a large α, the difference in results is noticeable. Further investigation, particularly with lab test, is warranted.

References

[1] J.T. Oden, T. Belytschko, I. Babuska and T.J.R. Hughes, "Research directions in computational mechanics," *Comput. Methods Appl. Mech, Engrg*., v. 192, p. 913-922, 2003.

[2] T. Belytschko, W.K. Liu and B. Moran, "Nonlinear finite elements for continua and structures". John Wiley & Sons, Ltd., Chichester, England, 2000.

[3] J.T. Oden, private communication, 2002.